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#### ORTHOTROPIC PLATE WITH INCLUSION HEATED BY A HEAT SOURCE

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UDC 536.24

The article presents solutions of steady problems of heat conduction for an orthotropic plate with foreign inclusion of arbitrary and small thickness.

We consider an orthotropic plate with thickness  $2\delta$  with an inclusion in the form of a strip of width  $2h$ . We represent the thermophysical characteristics of the system under examination in the form

$$p(x) = p^{(1)} + (p^{(0)} - p^{(1)})N(x), \quad (1)$$

where  $p^{(0)}$  and  $p^{(1)}$  are the characteristics of the inclusion and of the base material, respectively,  $N(x) = S_+(x+h) - S_-(x-h)$ ,  $S_{\pm}(\xi)$  are asymmetric unique functions [1]. Heat exchange with the environment is effected through the surfaces  $z = \pm\delta$  according to Newton's law. For determining the temperature we have the equation [2]

$$\frac{\partial}{\partial x} \left[ \lambda_x(x) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \lambda_y(x) \frac{\partial T}{\partial y} \right] - \frac{\alpha_z(x)}{\delta} T = -w. \quad (2)$$

Heating of a Plate by a Linear Heat Source. We assume that an infinite orthotropic plate with an inclusion in the form of a strip  $2h$  wide is heated by a linear heat source of intensity  $q$ , situated at the center of the inclusion. To determine the stationary temperature field, we have Eq. (2), where  $w = \frac{q}{2\delta} \delta(x)\delta(y)$ , and the boundary conditions

$$\lim_{|x| \rightarrow \infty} T = 0. \quad (3)$$

Taking (1) into account and using the formula

$$(\varphi\psi)' = \varphi'\psi + \varphi\psi' \mp [\varphi][\psi]\delta_{\pm}(x-x_1), \quad (4)$$

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Institute of Applied Problems of Mechanics and Mathematics, Academy of Sciences of the Ukrainian SSR, Lvov. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 50, No. 1, pp. 120-126, January, 1986. Original article submitted October 15, 1984.

we obtain

$$\begin{aligned} & \frac{\partial^2 T}{\partial x^2} + [k_y^{(1)} + (k_y^{(0)} - k_y^{(1)}) N(x)] \frac{\partial^2 T}{\partial y^2} + (1 - K_x^{-1}) \times \\ & \times \left[ \frac{\partial T}{\partial x} \Big|_{x=-h-0} \delta_+(x+h) - \frac{\partial T}{\partial x} \Big|_{x=h+0} \delta_-(x-h) \right] - \\ & - [\alpha_1^2 + (\alpha_0^2 - \alpha_1^2) N(x)] T = -Q \delta(x) \delta(y), \end{aligned} \quad (5)$$

where  $[\varphi]$ ,  $[\psi]$  are the jumps of the functions  $\varphi$  and  $\psi$ , respectively, at the point  $x_1$ ,  $k_y^{(n)} = \lambda_y^{(n)}/\lambda_x^{(n)}$ ,  $\alpha_n^2 = \alpha_z^{(n)}/\delta\lambda_x^{(n)}$  ( $n = 0, 1$ );  $K_x = \lambda_x^{(0)}/\lambda_x^{(1)}$ ;  $Q = q/2\delta\lambda_x^{(0)}$ ;  $\delta_{\pm}(\xi) = S_{\pm}(\xi)$ .

We multiply Eq. (5) by  $N(x)$  and introduce the substitution [3, 4]

$$\Theta = TN(x). \quad (6)$$

Then for determining the function  $\theta$  we obtain the equation

$$\begin{aligned} & \frac{\partial^2 \Theta}{\partial x^2} + k_y^{(0)} \frac{\partial^2 \Theta}{\partial y^2} - \alpha_0^2 \Theta = K_x^{-1} \left[ \frac{\partial T}{\partial x} \Big|_{x=-h-0} \delta_+(x+h) - \right. \\ & \left. - \frac{\partial T}{\partial x} \Big|_{x=h+0} \delta_-(x-h) \right] + T \Big|_{x=-h-0} \delta'_+(x+h) - \\ & - T \Big|_{x=h+0} \delta'_-(x-h) - Q \delta(x) \delta(y). \end{aligned} \quad (7)$$

Applying the integral Fourier transformation with respect to  $y$  to (7), we write

$$\begin{aligned} & \frac{d^2 \bar{\Theta}}{dx^2} - \gamma_0^2 \bar{\Theta} = K_x^{-1} (T_- - T_+) + (1 - K_x^{-1}) (\bar{T} \Big|_{x=-h-0} \delta'_+(x+h) - \\ & - \bar{T} \Big|_{x=h+0} \delta'_-(x-h)) - \frac{Q}{\sqrt{2\pi}} \delta(x), \end{aligned} \quad (8)$$

where  $T_- = \bar{T}(-x-2h) \delta'_+(x+h)$ ,  $T_+ = \bar{T}(-x+2h) \delta'_-(x-h)$ ,  $\gamma_0^2 = k_y^{(0)} \eta^2 + \alpha_0^2$ . The solution of this equation is as follows:

$$\bar{\Theta} = K_x^{-1} (P_1^- - P_1^+) + (1 - K_x^{-1}) (P_2^- - P_2^+) - \frac{Q}{\sqrt{2\pi}} \frac{\text{sh } \gamma_0 x}{\gamma_0} S(x), \quad (9)$$

where

$$\begin{aligned} P_1^{\pm} &= \frac{1}{\gamma_0} \left\{ \frac{d}{d\xi} [\bar{T}(\pm 2h - \xi) \text{sh } \gamma_0 (\xi - x)] \Big|_{\xi = \pm h \mp 0} \right\} S_{\mp}(x \mp h); \\ P_2^{\pm} &= \bar{T} \Big|_{x = \pm h \pm 0} \text{ch } \gamma_0 (x \mp h) S_{\mp}(x \mp h); \quad S(x) = \begin{cases} 1, & x > 0, \\ 0.5, & x = 0, \\ 0, & x < 0. \end{cases} \end{aligned}$$

Taking (9) into account, we have the following equation for determining  $\bar{T}$ :

$$\begin{aligned} & \frac{d^2 \bar{T}}{dx^2} - \beta^2 \bar{T} = K_x^{-1} (\gamma_0^2 - \beta^2) (P_1^- - P_1^+) + (1 - K_x^{-1}) (\gamma_0^2 - \beta^2) (P_2^- - P_2^+) - \\ & - \frac{Q(\gamma_0^2 - \beta^2)}{\sqrt{2\pi}} \frac{\text{sh } \gamma_0 x}{\gamma_0} S(x) + (K_x^{-1} - 1) \left[ \frac{d\bar{T}}{dx} \Big|_{x=-h-0} \delta_+(x+h) - \right. \\ & \left. - \frac{d\bar{T}}{dx} \Big|_{x=h+0} \delta_-(x-h) \right] - \frac{Q}{\sqrt{2\pi}} \delta(x), \end{aligned} \quad (10)$$

where  $\beta^2 = k_y^{(1)} \eta^2 + \alpha_1^2$ .

Applying the Fourier transformation with respect to  $y$  to the boundary conditions (3), we obtain

$$\lim_{|x| \rightarrow \infty} \bar{T} = 0. \quad (11)$$

The solution of Eq. (10) with the boundary conditions (11) taken into account has the form

$$\bar{T} = \frac{Q}{2\sqrt{2\pi}} \left\{ k_1 [S_-(-x-h) \exp[\beta(x+h)] + S_-(x-h) \times \exp[-\beta(x-h)]] + (k_1 k_2 \operatorname{ch} \gamma_0 x - \operatorname{sh} \gamma_0 |x|) \frac{N(x)}{\gamma_0} \right\}, \quad (12)$$

where  $k_1 = (K_x^{-1} \beta \operatorname{ch} \gamma_0 h + \gamma_0 \operatorname{sh} \gamma_0 h)^{-1}$ ;  $k_2 = K_x^{-1} \beta \operatorname{sh} \gamma_0 h + \gamma_0 \operatorname{ch} \gamma_0 h$ . Going over in (12) from the transforms to the originals, we obtain

$$T = \frac{Q}{2\pi} \int_0^{\infty} \cos \eta y \left\{ k_1 [S_-(-x-h) \exp[\beta(x+h)] + S_-(x-h) \times \exp[-\beta(x-h)]] + (k_1 k_2 \operatorname{ch} \gamma_0 x - \operatorname{sh} \gamma_0 |x|) \frac{N(x)}{\gamma_0} \right\} d\eta.$$

If the inclusion is thin ( $h \ll \delta$ ), then with the equalities [5, 6] taken into account

$$\lim_{h \rightarrow 0} \frac{N(x)}{2h} = \delta(x), \quad \delta'(x) f|_{x=0} = \frac{1}{2} \delta'(x) [f|_{x=-0} + f|_{x=+0}], \quad (13)$$

Eq. (2) assumes the form

$$\frac{\partial^2 T}{\partial x^2} + k_y^{(1)} \frac{\partial^2 T}{\partial y^2} - \kappa_1^2 T + 2h\delta(x) (k_y^{(0)} K_x - k_y^{(1)}) \frac{\partial^2 T}{\partial y^2|_{x=0}} - 2h\delta(x) ((\kappa_0^*)^2 - \kappa_1^2) T|_{x=0} = -Q^* \delta(x) \delta(y), \quad (14)$$

where  $(\kappa_0^*)^2 = \alpha_z^{(0)}/\delta\lambda_x^{(1)}$ ;  $Q^* = q/2\delta\lambda_x^{(1)}$ .

Applying the Fourier transformation with respect to  $y$  to Eq. (14), we have

$$\frac{d^2 \bar{T}}{dx^2} - \beta^2 \bar{T} = \overline{\Omega(y)} \delta(x), \quad (15)$$

where

$$\overline{\Omega(y)} = -\frac{Q^*}{\sqrt{2\pi}} + 2h\bar{T}|_{x=0} [(\kappa_0^*)^2 - \kappa_1^2 + \eta^2 (k_y^{(0)} K_x - k_y^{(1)})].$$

We write the solution of Eq. (15) with (11) taken into account:

$$\bar{T} = \frac{Q^* \exp(-\beta|x|)}{2\sqrt{2\pi}} \left( \frac{a_1}{\beta - \gamma_1^*} + \frac{a_2}{\beta - \gamma_2^*} \right), \quad (16)$$

where

$$a_1 = -a_2 = \left[ 1 - \frac{4h^2}{\delta\lambda_x^{(1)}} (K_y - 1) (\alpha_z^{(0)} - \alpha_z^{(1)} K_y) \right]^{-1/2};$$

$$\gamma_{1,2}^* = \frac{-1 \pm a_1^{-1}}{2h(K_y - 1)}; \quad K_y = \lambda_y^{(0)}/\lambda_y^{(1)}.$$

Going over in (16) from the transforms to the originals, we obtain

$$T = \frac{Q^* a_1}{2\pi} \int_0^{\infty} \exp(-\beta|x|) \cos \eta y \left( \frac{1}{\beta - \gamma_1^*} - \frac{1}{\beta - \gamma_2^*} \right) d\eta.$$

Heating a Plate by a System of Heat Sources. Let an infinite orthotropic plate with an inclusion in the form of a strip of width  $2h$  be heated by a system of equally spaced concentrated heat sources with density

$$w = \frac{q}{2\delta} \frac{\delta(x)}{2c} \left( 1 + 2 \sum_{m=1}^{\infty} \cos \lambda_m y \right), \quad \lambda_m = \frac{m\pi}{c}, \quad (17)$$

where  $2c$  is the distance between neighboring heat sources. To determine the stationary temperature field we have Eq. (2) and the boundary conditions (3). Using (4), we write Eq. (2) as follows:

$$\begin{aligned} \frac{\partial^2 T}{\partial x^2} + [k_y^{(1)} + (k_y^{(0)} - k_y^{(1)})N(x)] \frac{\partial^2 T}{\partial y^2} + (1 - K_x^{-1}) \left[ \frac{\partial T}{\partial x|_{x=-h-0}} \delta_+(x+h) - \right. \\ \left. - \frac{\partial T}{\partial x|_{x=h+0}} \delta_-(x-h) \right] - [\alpha_1^2 + (\alpha_0^2 - \alpha_1^2)N(x)] T = \\ = - \frac{Q}{2c} \delta(x) \left( 1 + 2 \sum_{m=1}^{\infty} \cos \lambda_m y \right). \end{aligned} \quad (18)$$

Multiplying (18) by  $N(x)$  and introducing the substitution (6), we arrive at the equation

$$\begin{aligned} \frac{\partial^2 \Theta}{\partial x^2} + k_y^{(0)} \frac{\partial^2 \Theta}{\partial y^2} - \alpha_0^2 \Theta = K_x^{-1} \left[ \frac{\partial T}{\partial x|_{x=-h-0}} \delta_+(x+h) - \right. \\ \left. - \frac{\partial T}{\partial x|_{x=h+0}} \delta_-(x-h) \right] + T|_{x=-h-0} \delta'_+(x+h) - T|_{x=h+0} \delta'_-(x-h) - \\ - \frac{Q}{2c} \delta(x) \left( 1 + 2 \sum_{m=1}^{\infty} \cos \lambda_m y \right). \end{aligned} \quad (19)$$

When we apply the Fourier transformation with respect to  $y$  to (19) and solve the obtained equation, we have

$$\begin{aligned} \bar{\Theta} = K_x^{-1} (P_1^- - P_1^+) + (1 - K_x^{-1}) (P_2^- - P_2^+) - \frac{Q}{c} \frac{\text{sh } \gamma_0 x}{\gamma_0} \sqrt{\frac{\pi}{2}} \left\{ \delta(\eta) + \right. \\ \left. + \sum_{m=1}^{\infty} [\delta(\eta + \lambda_m) + \delta(\eta - \lambda_m)] \right\} S(x). \end{aligned} \quad (20)$$

Using (20), we determine the function  $\bar{T}$  from the equation

$$\begin{aligned} \frac{d^2 \bar{T}}{dx^2} - \beta^2 \bar{T} = K_x^{-1} (\gamma_0^2 - \beta^2) (P_2^- - P_1^+) + (1 - K_x^{-1}) (\gamma_0^2 - \beta^2) (P_2^- - P_2^+) + \\ + (K_x^{-1} - 1) \left[ \frac{d\bar{T}}{dx|_{x=-h-0}} \delta_+(x+h) - \frac{d\bar{T}}{dx|_{x=h+0}} \delta_-(x-h) \right] - \frac{Q}{c} \sqrt{\frac{\pi}{2}} \left\{ \delta(\eta) + \right. \\ \left. + \sum_{m=1}^{\infty} [\delta(\eta + \lambda_m) + \delta(\eta - \lambda_m)] \right\} \left[ (\gamma_0^2 - \beta^2) \frac{\text{sh } \gamma_0 x}{\gamma_0} S(x) + \delta(x) \right]. \end{aligned} \quad (21)$$

The solution of Eq. (21), with (11) taken into account, have the form

$$\begin{aligned} \bar{T} = \frac{Q}{2c} \sqrt{\frac{\pi}{2}} \left\{ \delta(\eta) + \sum_{m=1}^{\infty} [\delta(\eta + \lambda_m) + \delta(\eta - \lambda_m)] \right\} \times \\ \times \left\{ k_1 [S_-(-x-h) \exp[\beta(x+h)] + S_-(x-h) \exp[-\beta(x-h)]] + \right. \\ \left. + \frac{N(x)}{\gamma_0} (k_1 k_2 \text{ch } \gamma_0 x - \text{sh } \gamma_0 |x|) \right\}. \end{aligned} \quad (22)$$

Going over in (22) from the transforms to the originals, we obtain

$$T = \frac{Q}{4c} \left\langle \psi(\kappa_0, \kappa_1) + \frac{N(x)}{\kappa_0} [\varphi(\kappa_0, \kappa_1) \operatorname{ch} \kappa_0 x - \operatorname{sh} \kappa_0 |x|] + \right. \\ \left. + 2 \sum_{m=1}^{\infty} \cos \frac{m\pi y}{c} \left\{ \psi(\gamma_m^{(0)}, \gamma_m^{(1)}) + \right. \right. \\ \left. \left. + \frac{N(x)}{\gamma_m^{(0)}} [\varphi(\gamma_m^{(0)}, \gamma_m^{(1)}) \operatorname{ch} \gamma_m^{(0)} x - \operatorname{sh} \gamma_m^{(0)} |x|] \right\} \right\rangle, \quad (23)$$

where

$$\varphi(a, b) = \frac{a + bK_x^{-1} + \exp(-2ah)(a - bK_x^{-1})}{a + bK_x^{-1} - \exp(-2ah)(a - bK_x^{-1})}; \\ \psi(a, b) = \frac{S_-(-x-h)\exp[b(x+h)] + S_-(x-h)\exp[-b(x-h)]}{K_x^{-1}b \operatorname{ch} ah + a \operatorname{sh} ah}; \\ \gamma_m^{(n)} = \sqrt{k_y^{(n)} \frac{m^2 \pi^2}{c^2} + \kappa_n^2} \quad (n = 0, 1).$$

In the case of a thin inclusion we have the equation of heat conduction

$$\frac{d^2 \bar{T}}{dx^2} - \beta^2 \bar{T} = \delta(x) \left\langle -\frac{Q^*}{c} \sqrt{\frac{\pi}{2}} \left\{ \delta(\eta) + \sum_{m=1}^{\infty} [\delta(\eta + \lambda_m) + \delta(\eta - \lambda_m)] \right\} + \right. \\ \left. + 2h\bar{T}|_{x=0} [(k_y^{(0)} K_x - k_y^{(1)}) \eta^2 + (\kappa_0^*)^2 - \kappa_1^2] \right\rangle, \quad (24)$$

whose solution with conditions (11) is

$$\bar{T} = \frac{Q^*}{2c} \sqrt{\frac{\pi}{2}} \left\{ \frac{\exp(-\kappa_1 |x|)}{\kappa_1 + h[(\kappa_0^*)^2 - \kappa_1^2]} \delta(\eta) + \sum_{m=1}^{\infty} \exp(-|x| \sqrt{k_y^{(1)} \lambda_m^2 + \kappa_1^2}) \times \right. \\ \left. \times \frac{\delta(\eta + \lambda_m) + \delta(\eta - \lambda_m)}{\sqrt{k_y^{(1)} \lambda_m^2 + \kappa_1^2 + h[(\kappa_0^*)^2 - \kappa_1^2 + \lambda_m^2 (k_y^{(0)} K_x - k_y^{(1)})]}} \right\}. \quad (25)$$

Going over in (25) from the transforms to the originals, we obtain

$$T = \frac{Q^*}{4c} \left\{ \frac{\exp(-\kappa_1 |x|)}{\kappa_1 + h[(\kappa_0^*)^2 - \kappa_1^2]} + 2 \sum_{m=1}^{\infty} \cos \lambda_m y \times \right. \\ \left. \times \frac{\exp(-|x| \sqrt{k_y^{(1)} \lambda_m^2 + \kappa_1^2})}{\sqrt{k_y^{(1)} \lambda_m^2 + \kappa_1^2 + h[(\kappa_0^*)^2 - \kappa_1^2 + \lambda_m^2 (k_y^{(0)} K_x - k_y^{(1)})]}} \right\}. \quad (26)$$

We used a computer ES-1022 to carry out calculations by formulas (23) and (26) for a plate of glass textolite KAST-V and an inclusion of steel brand 1Kh18N9T. The results of the investigations of the dimensionless temperature  $\vartheta = T \frac{8\delta\lambda_x^{(1)}}{q}$  in a plate heated by a system of concentrated heat sources are presented in Figs. 1 and 2. The solid curves in Fig. 1a, b correspond to the temperature in a plate with a striplike inclusion, the dashed lines relate to a narrow inclusion. The values of the parameters are as follows:  $Bi_0 = \alpha_2^{(0)}\delta/\lambda_x^{(1)} = 1$ ;  $Bi_1 = \alpha_2^{(1)}\delta/\lambda_x^{(1)} = 0$ ;  $0,01; 0,1$ ;  $c/\delta = 100$ ;  $h/\delta = H = 0,1$ .

It follows from Fig. 1a that at the points  $y = \pm(2k+1)100\delta = \pm(2k+1)c$ ,  $k = 0, 1, 2, \dots$ , the temperature attains its minimum. In Fig. 1b the curves of the change of temperature along the axis of abscissas are shown. We note that in the inclusion the temperature changes imperceptibly compared with the base material of the plate for a system with striplike inclusion. With increasing heat transfer from the surface of the base material the temperature drops.

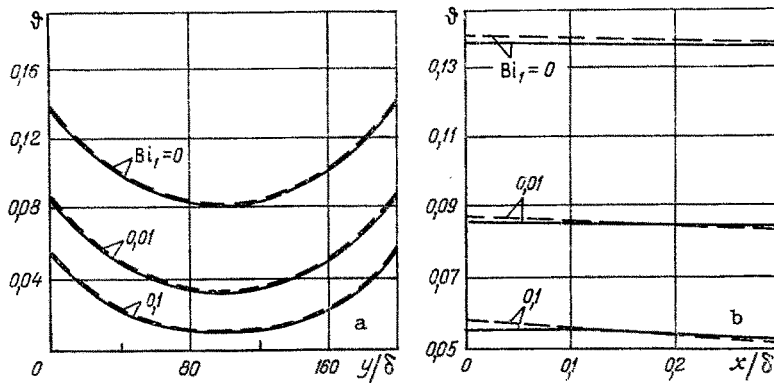


Fig. 1. Change of the dimensionless temperature  $\phi$  along the axis of the inclusion (a) and along the axis of abscissas (b).

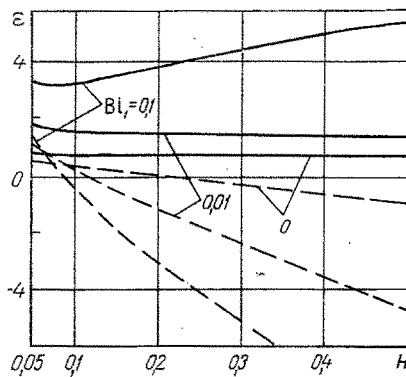


Fig. 2. Change of the relative error  $\varepsilon$  in dependence on the parameter  $H$  at the points  $x = 0; h; y = \pm 200 k\delta$ ,  $k = 0, 1, 2, \dots$

In Fig. 2 we see the dependences of the relative error  $\varepsilon = (T_{nr} - T_{str})/T_{str} \cdot 100\%$  on the parameter  $H$ . The solid curves correspond to the error at the point  $x = 0, y = 0$ , the dashed lines to the point  $x = h, y = 0$ . Numerical investigations show that an increase of the parameter  $H$  and of heat transfer from the surface of the base material lead to an increase of the error of temperature determined according to the model of a thin inclusion.

#### NOTATION

$T$ , temperature;  $x, y$ , Cartesian coordinates;  $\delta$ , half-thickness of the plate;  $h$ , half-width of the inclusion;  $c$ , half the distance between the heat sources;  $w$ , power of the heat sources;  $\lambda_x, \lambda_y$ , thermal conductivity in the directions of the  $Ox$  and  $Oy$  axes;  $\alpha_z$ , heat-transfer coefficient from the surface  $z = \pm\delta$ ;  $B$ , Biot number;  $\delta(\xi)$ , Dirac delta function;  $S\pm(\xi)$ , asymmetric unit functions;  $S(\xi)$ , symmetric unit function.

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## CALCULATION OF TOMOGRAPHIC PROJECTIONS

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UDC 517.39

The article suggests a method of calculating tomographic projections.

The problem of interaction between x rays and the substance of the investigated object, arising in the field of computerized tomography, reduces to the calculation of tomographic projections [1]. The present article submits a method of calculating parallel and bundle tomographic projections for one class of images of the section of the object; the terminology and some of the designations are taken over from [1].

Let  $w$  and  $\hat{w}$  be the applicates of points of the plane of the object's section in the initial system  $x, y$  and in the system of coordinates  $\hat{x}, \hat{y}$  rotated through the angle  $\theta$ , respectively,  $\hat{w} = we^{-i\theta}$ ; let  $\mu(x, y)$  and  $\mu_0(x, \hat{y})$  be the distribution of the absorption coefficient by the material of the object in the initial and in the rotated system of coordinates, respectively,  $\mu_0(\hat{x}, \hat{y}) = \mu(x, y)$ ; the function  $\mu(x, y)$  is called the image of the section of the object. Then for x rays passing along the straight line  $\hat{x} = \text{const}$ , the logarithm of the ratio of its intensity at the entrance into the object to the intensity at the exit from the object, called the parallel tomographic projection  $p_0(\hat{x})$  of the section, is determined by the formula

$$p_0(\hat{x}) = \int_{-\infty}^{+\infty} \mu_0(\hat{x}, \hat{y}) d\hat{y}. \quad (1)$$

Assume that from the source lying at the point  $\rho \exp \left[ i \left( \beta - \frac{\pi}{2} \right) \right]$  there emerges a beam in the direction parallel to the vector  $\exp \left[ i \left( \frac{\pi}{2} + \beta + \gamma \right) \right]$ ; the logarithm of the ratio of its intensities at the entrance into and at the exit from the object is called the bundle projection  $h_\beta(\gamma)$  of the section; it is correlated with the parallel projection by the relation [1]

$$h_\beta(\gamma) = p_{0(\beta, \gamma)}(\hat{x}(\beta, \gamma)), \quad (2)$$

where the dependences  $\hat{x}(\beta, \gamma)$ ,  $\theta(\beta, \gamma)$  have the form

$$\hat{x} = -\rho \sin \gamma, \quad \theta = \beta + \gamma. \quad (3)$$

We introduce the notation:  $l, n$  are integers,  $n = 1, 2, \dots, N$ ;  $l = 1, 2, \dots, L_n$ ;  $g(n, l)$  is the region bounded by an ellipse with the center at the point  $R(n, l) \exp[i\varphi(n, l)]$ , the semi-axis  $a(n, l)$ ,  $b(n, l)$ , the first of which is inclined to the radius vector of the center of the ellipse at the angle  $\Phi(n, l)$ ; if for some  $n_0, l_0$  we have  $R(n_0, l_0) = 0$ , then we put  $\varphi(n_0, l_0) = 0$ ;  $g(0)$  is the region bounded by an ellipse with the center at the origin of coordinates, semiaxes  $a(0)$ ,  $b(0)$ , the first of which is inclined to the x axis at the angle  $\Phi(0)$ .

Let us examine the class of images  $\mu(x, y)$  for which the following condition is fulfilled; the section of the object is the domain  $g(0)$ ;  $g(n, l) \subset g(0)$  for all  $n, l$ ; the sets  $G(n)$ , determined by the relation

$$G(n) = \bigcup_{l=1}^{L_n} g(n, l), \quad (4)$$